A Sensitivity Analysis of Centrifugal Compressors' Empirical Models

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The mean-line method using empirical models is the most practical method of predicting off-design performance. To gain insight into the empirical models, the influence of empirical models on the performance prediction results is investigated. We found that, in the two-zone model, the secondary flow mass fraction has a considerable effect at high mass flow-rates on the performance prediction curves. In the TEIS model, the first element changes the slope of the performance curves as well as the stable operating range. The second element makes the performance curves move up and down as it increases or decreases. It is also discovered that the slip factor affects pressure ratio, but it has little effect on efficiency. Finally, this study reveals that the skin friction coefficient has significant effect on both the pressure ratio curve and the efficiency curve. These results show the limitations of the present empirical models, and more resonable empirical models are reeded.

Key Words: Centrifugal Compressor, Off-Design Performance Prediction, Two-zone Model, TEIS Model, Slip Factor, Loss Model, Sensitivity, Vaneless Diffuser, Skin Friction Coefficient

Nomenclature

C : Absolute velocity
C_p : Pressure recovery coefficient
c_f : Friction loss coefficient of vaneless diffuser
DR : Diffusion ratio in impeller (W_{1t}/W_{2p})
W : Relative velocity
Z : Number of blade
β : Blade angle (from tangential direction)
δ : Deviation angle
ε : Secondary flow area fraction
η : Effectiveness in TEIS model
ρ : Density
σ : Slip factor
τ_w : Shear stress in vaneless diffuser
χ : Secondary flow mass fraction

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Subscripts

1 : Impeller inlet
2 : Impeller exit
a : First element in TEIS model
b : Second element in TEIS model, blade
i : Ideal state
p : Primary zone in Two-zone model
s : Secondary zone in Two-zone model
t : Tip
th : Throat

1. Introduction

Historically, compressor design and prediction methods have been based on empirical models and the empirical parameters. Despite the impressive progress in computational fluid dynamics, the mean-line method using empirical models has been considered the most practical method of predicting the off-design performance of turbomachinery. Much research effort has been

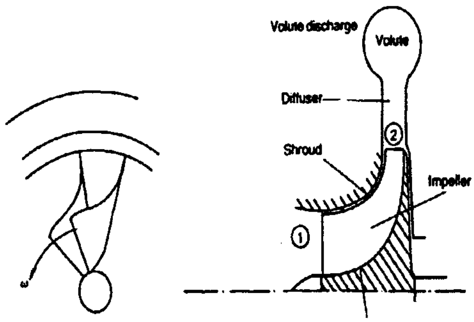

Fig. 1 Schematics of Centrifugal Compressor

exerted to develope and refine empirical models (e.g. Dean(1972), Galvas(1973), Jansen(1990), Aungier(1995), Oh et al(1997) and Yoon and Baek(2000)). Better understanding of empirical models and their parameters would contribute to developing a more accurate performance prediction method.

The off-design performance is defined as the performance of the compressor at flow conditions and speeds other than that for which the compressor was designed. The off-design performance prediction differs from the design problem in that the compressor geometry is given and the Objective is to find the compressor outlet conditions for a range of speeds and mass flow-rates. Specifically, the variations in efficiency and pressure ratio along the operating line are needed. This problem of off-design performance prediction is one of the most difficult tasks facing compressor designers. Therefore, the main purpose of this paper is to investigate the empirical models to predict centrifugal compressor performance at off-design conditions. A centrifugal compressor consists of an impeller, which imparts a high velocity to air, and a vaneless diffuser where the air is decelerated with a consequent rise in static pressure. Figure 1 is a meridional sketch of a centrifugal compressor.

To analyze the impeller, we adopted the Two-zone model, the TEIS model, and the slip factor model. The combination of the Two-zone and the TEIS model gives a modified mean-line method based on the jet/wake flow discovered by Eckardt (1976). Our modified method considers two streamlines without any blockage or internal loss whereas the classical mean-line method considers only one streamline with blockage and internal loss. Japikse (1985) and Oh et al (1999) showed that the use of modified methods results in more accurate predictions. However, the Two-zone and the TEIS model employ some empirical parameters which have not yet been understood thoroughly. These parameters are the secondary flow mass fraction, secondary flow deviation angle, and two effectiveness parameters-η_a and η_b. Thus, in this study, the influence of these parameters on the overall performance prediction is investigated. Slip factor is also important in analyzing impellers. Although much has been written with regard to the slip factor, there is an urgent need to validate various existing models at off-design condition. Therefore, this study compares slip factor models with experimental data at off-design conditions and examines their effect on performance prediction.

To analyze vaneless diffusers, Stanitz's (1952) equation is applied. we investigate its effect on the pressure recovery and overall performance.

2. Centrifugal Impeller

2.1 Two-zone model

Eckardt (1976) first performed detailed laser measurements of complex flows in centrifugal impellers. In his results, after one half of the impeller, flow separation occurs and this leads to a highly distorted jet/wake velocity patterns. Based on this investigation, Japikse (1985) developed a mathematically simple but physically reasonable flow model. He simplified the internal flow by two regions (Fig. 2)-the primary region and the wake region. A comprehensive description of this model is given in Japikse (1985) and Yoon and Baek (2000). This model employs two important empirical parameters-the secondary flow mass fraction ($\chi = m_s/m$) and the secondary flow deviation angle (δ_{2s}).

Classically, a constant value of χ, regardless of the mass flow-rate, is adopted by Dean (1972), Whitfield (1990), and Japikse (1996). However, the exact value of χ is controversial. Recently Oh et al. (1999) suggested χ as a function of secon-

Table 1 Recommended values of parameters for TEIS model and secondary mass flowrate (Japikse, 1996)

Case	χ	η_a	η_b
Large, well-designed rotors	0.1-0.2	0.9-1.1	0.4 to 0.6
(>10″ to 12″D or smaller if well-designed)			
Medium size, well-designed	0.15-0.25	0.8-0.9	0.3 to 0.5
(4″ to 10″D)			
Medium size, ordinary design	0.20-0.30	0.6-0.8	0.0 to 0.4
(4″ to 10″ D)			
small or poorly designed	0.25-0.35	0.4-0.6	-0.3 to 0.3
(<4″D)			

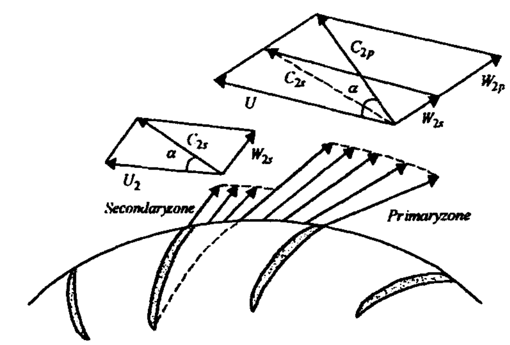

Fig. 2 Schematics of two-zone model

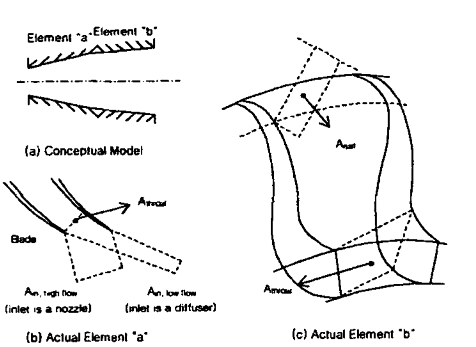

Fig. 3 Two-element-in-series conceptual model

dary flow area fraction ($\varepsilon = A_s/A_2$) as shown below.

$$\chi = 0.93\varepsilon^2 + 0.09\varepsilon \tag{1}$$

In this paper, we examine the influence of secondary flow mass fraction on off-design prediction. For secondary flow deviation angle (δ_{2s}), Whitfield (1990) assumed $\delta_{2s}=0$ and Japikse (1985) assumed that δ_{2s} has little effect on off-design prediction. To check these assumptions, the influence of δ_{2s} on performance is investigated.

2.2 TEIS model

In the TEIS model of Japikse (1996), developed to predict the diffusion ratio in the primary zone, the passage of the impeller is regarded as Two Element In Series as shown in Fig. 3. The first element covers the region between the inducer and the impeller throat, which may be act as either a diffuser or a nozzle depending on the incidence angle. The second element is the part from the impeller throat to the impeller exit, which is a constant geometrical diffuser. With this model, the following Eqs. (2) can be derived to find the diffusion ratio.

$$DR_2 = \frac{W_{1t}}{W_{2p}} \tag{2a}$$

$$DR_2^2 = \frac{1}{1-\eta_a C_{pa.i}} \times \frac{1}{1-\eta_b C_{pb.i}} \tag{2b}$$

This equations contain two empirical parameters-η_a and η_b corresponding to the first and the second element in TEIS model, respectively. Japikse (1996) classified the range of the values of η_a, η_b and χ, according to the impeller geometry as shown in Table 1. The cmpeller geometry. The effect of η_a and η_b on off-design

Table 2 Specification of Eckardt centrifugal compressors

Type	β_{1t}(deg)	β_2(deg)	r_{1t}(mm)	r_2(mm)	r_3(mm)	b_2(mm)	Z
Eckardt O	-63	0	140	200	400	26	20
Eckardt A	-63	-30	140	200	400	26	20
Cho et al's	-61.48	-48.83	36.805	60	117	7.826	14

prediction performance is carried out.

2.3 Slip factor model

The slip factor model should also be taken into account to predict centrifugal impeller performance because the slip factor model is essential to estimate the real work input by centrifugal impellers. The most popular slip factor models can be summarized as follows.

Stodola (1927) :

$$\sigma=1-\frac{\pi}{Z}\sin\beta_{2b} \tag{3}$$

Stanitz (1952) :

$$\sigma=1-0.63\frac{\pi}{Z} \tag{4}$$

Wiesner (1967) :

$$\sigma=1-\left(\frac{\sqrt{\sin\beta_{2b}}}{Z^{0.7}}\right) \tag{5}$$

Paeng and Chung (2000) :

$$\sigma=1-fa \tag{5}$$

where

$$f=0.853+0.025\exp((90-\beta_{2b})/24) \tag{5a}$$

with either

$$a=\frac{\sin\left(\frac{\pi}{Z}\sin\beta_{2b}\right)}{1+\sin\left(\frac{\pi}{Z}\sin\beta_{2b}\right)} \tag{5b}$$

or

$$a=\frac{1-\exp\left(-\frac{2\pi}{Z}\sin\beta_{2b}\right)}{2} \tag{5c}$$

Currently, one of these models is applied for preliminary design and off-design performance prediction. Therefore, the effect of the slip factor model on the overall performance curves is investigated.

3. Skin Friction Coefficient in Vaneless Diffuser

For vaneless diffusers, the present authors (2000) simplified Stanitz's (1952) equations which can be solved by 4-stage Runge-Kutta method. These equations include one empirical parameter-skin friction coefficient (c_f) defined by Eq. (6)

$$c_f=\frac{2\tau_w}{\rho C^2} \tag{6}$$

For this parameter, Japikse (1996) proposed an empirical equation (Eq. (7)), and he recommended 0.01 for the value of k.

$$c_f=k\left(\frac{180{,}000}{Re}\right)^{0.2} \tag{7}$$

We studied the influence of k on the overall performance and pressure recovery at off-design conditions.

4. Result

Using the empirical models described in the preceding sections, a mean-line performance prediction method is developed and implemented into a computer program with GUI (Graphic User Interface). The empirical models have been validated against experimental data of Eckardt O (1976), Eckardt A (1980), and Cho et al (1998). Table 2 lists the overall dimensions of these compressors. The optimal combination of the empirical models and the empirical parameters, to predict the off-design performance for the respective compressor, are found by trial and error and are designated as reference values. Off-design performance predictions are compared with experimental data in Figs. 4~6 and the agreement is quite good.

To evaluate the models, a detailed investigation

Table 3 Variations of the empirical models for the Eckardt O and the Eckardt A compressor

	Reference	Case 1	Case 2	Case 3
χ	0.15	0.1	0.2	Eq. (1)
(η_a, η_b)	(1.0, 0.5)	(0.8, 0.5)	(1.0, 0.25)	X
Slip factor	Wiesner	Stantiz	Stodola	Paeng and Chung
k	0.008	0.005	0.012	X

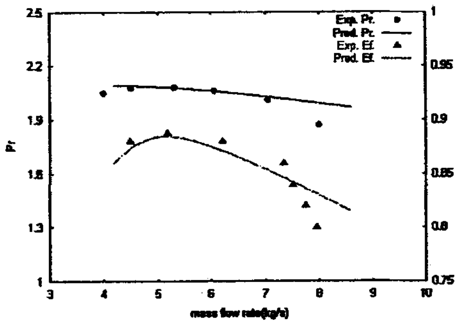

Fig. 4 Eckardt O compressor

of off-design performance curves, with variation of each empirical model in turn, is performed. The values of empirical models and parameters are listed in Table 3.

The sensitivity of the performance prediction curves with the change in secondary flow mass fraction (χ) is shown in Fig. 5. As χ decreases (case 1), the predicted pressure ratio and efficiency increase. Large χ means large losses and will cause the reduction of the pressure ratio and the efficiency. The influence of χ is especially considerable at high mass flow-rates. The use of Eq. (1), (case 3), results in a large discrepancy from the experimental data for Eckardt A compressor, implying that the recently proposed model needs more verification. To examine the effect of Eq. (1) in detail, the prediction curves of χ, at off-design conditions, is presented in Fig. 9. For Eckardt O compressor, χ is higher than the reference curve (χ=0.15) except at the high mass flow-rates. In contrast to this, for Eckardt A compressor, χ is predicted lower and give rise to the prediction of the higher pressure ratio and efficiency.

Figures 10 and 11 show the secondary flow

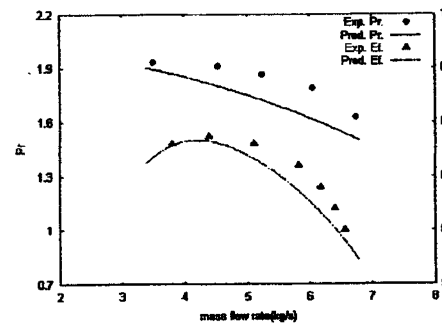

Fig. 5 Eckardt A compressor

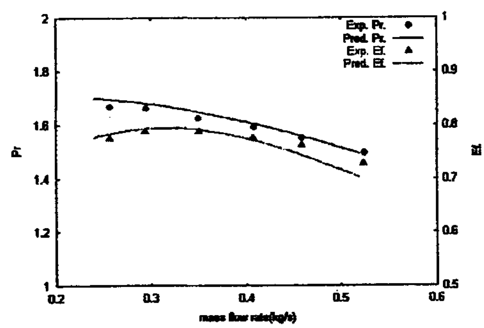

Fig. 6 Cho et al's compressor

area fraction (ε) over the operating range. For Eckadt O compressor, with a radial impeller, ε decrease as χ increases. On the other hand, for Eckardt A compressor, with a backswept impeller, convex shape curves are generated. Since there is insuffcient experimental or numerical result about the relationship between χ and ε, it is difficult to judge which modeling is more accurate. So, further experimental or numerical research on this subject is suggested, preferably in simple flow configurations (e. g. rotary bends) to gain complementary fundamental data for refining the Two-zone model. The influence of secondary flow deviation angle on performance prediction has also been investigated ; it has little or no effect on predictions.

Figures 12 and 13 show the effect of two elements' effectiveness, in TEIS model, on the prediction results. As the first element's effectiveness is decreased (case 1), the pressure ratio and efficiency decrease at low mass flow-rates and increase at high mass flow-rates. Also, the operating range is widened. On

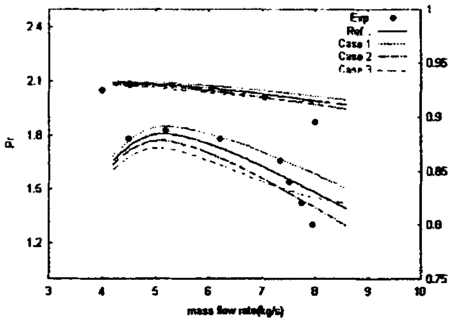

Fig. 7 Effect of Seconday flow mass-fraction on Eckardt O

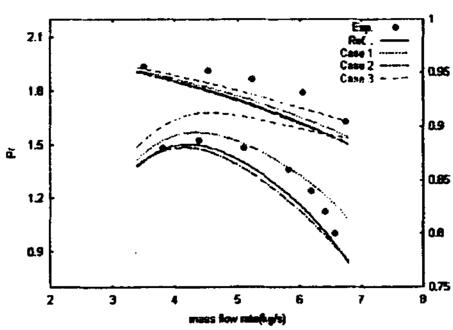

Fig. 8 Effect of Seconday flow mass-fraction on Eckardt A

the contrary, when the second element's effectiveness is decreased, the prediction results are decreased all over the whole range, without operating range extension. To examine the effect of TEIS model more thoroughly, we investigated the prediction of diffusion ratio as a function of mass flow-rate as shown in Fig. 14. Figure 14 shows that the decrease in the first element's effectiveness brings about a reduced increase in the diffusion ratio; decreases the mass flow-rate; and predicts surge at a lower flow-rate. However, the decrease in the second element's effectiveness merely lowers the diffusion ratio curve. In fact, the first element, as shown in Fig. 3, is effectively a variable geometry device which is very sensitive to mass flow-rate (i.e incidence) and is critical in predicting surge and choke points. On the other hand, since the second element is essentially a constant geometry device, it is not sensitive to mass flow-rate, and it has a little effect on surge and choke conditions.

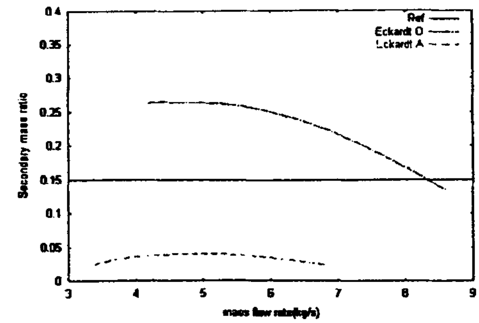

Fig. 9 Modeling of secondary flow mass-fraction

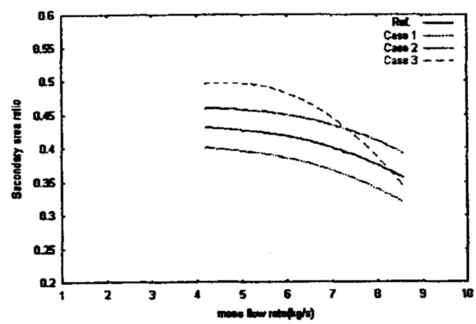

Fig. 10 Effect of secondary flow mass-fraction on the ratio of secondary area in Eckardt O

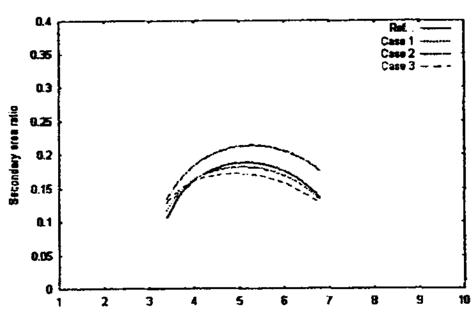

Fig. 11 Effect of secondary flow mass-fraction on the ratio of secondary area in Eckardt A

The comparison between predictions and experimental data for various slip factor values is conducted in Figs. 15~16. The experimental data for Eckardt O compressor, which has a radial impeller, rises as the flow-rate is reduced (i. e the flow deviates less and less from the blade direction). The coincidence of the experimental data with the Wiesner model and the Paneg and Chung's model is satisfactory. On the contrary, Eckardt A compressor, with a backswetp

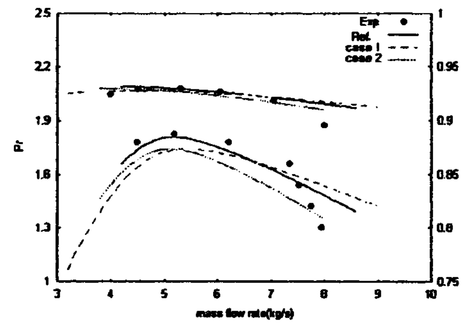

Fig. 12 Effect of TEIS model on Eckardt O

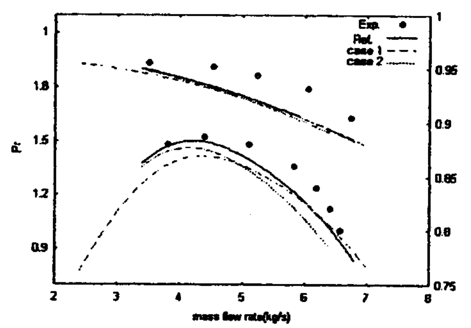

Fig. 13 Effect of TEIS model on Eckardt A

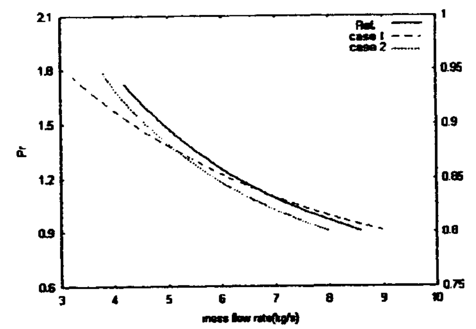

Fig. 14 Effect of TEIS model on diffusion ratio in Eckardt O

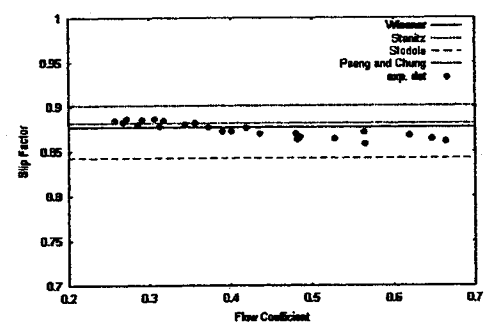

Fig. 15 Slip factor of Eckardt O

impeller, takes a completely different development, resulting in considerable deviation. Here the experimental data rise while the prediction results fall with increasing flow-rate. The limitation of the slip factor models is due to the fact that all the models were developed for the design point assuming inviscid flow and over simplified. In the future, additional consideration of the impeller blade shape will be mandatory for more reliable slip correlations.

Figures 17 and 18 show the slip factor effect on performance. The pressure ratio curves either rise or fall, according to the selected model The descending order is 'Stanitz', 'Paeng and Chung', ' Wiesner', and 'Stodola'. But the slip factor model has a little effect on the efficiency.

A set of performance prediction curves, while the variation of k in the skin friction coefficient model, are generated in Figs. 19~20. k has a significant effect on both pressure ratio and efficiency. The prediction curves rise with decreasing k. Japikse (1996) reported that k can vary from 0.005 to 0.02, depending on flow conditions and geometry. He recommended a value of 0.01. However, a slight change in k significantly affects prediction; therfore, a more detailed modeling about k (or c_f) is necessary. Figure 21 shows the c_f predicted by Eq. (7) with k=0.008. For Eckatdt O compressor, c_f retains almost constant values with a slight decrease at high mass flow-rates, while for Eckardt A compressor, it increases with increasing mass flow-rate. Figure 22 shows the distribution of pressure recovery coefficient along the wall of the vaneless diffuser at the design point, choke point, and surge point in Cho et al's compressor. The results show that k=0.012 is the most suitable for the design point, but k=0.017 and 0.016 are more accurate for choke and stall point, respectively.

Finally, the sensitivity of prediction results for Eckardt O compressor, are summarized in Table 4. The changes in pressure ratio and efficiency are represented in percentage according to the variation of the empirical models and the empirical parameters. As we can see in this table, k has a

Table 4 Effect of empirical parameters' variation on performance (in percentage)

model	variation	Stall pt. (4.2 kg/s)		Design pt. (5.3 kg/s)		Choke pt. (8.6 kg/s)	
		Pr.	Ef.	Pr.	Ef.	Pr.	Ef.
Ref.		0	0	0	0	0	0
Two-zone	$\chi-0.05$	0.05	0.47	0.53	0.79	1.47	2.33
	$\chi+0.05$	−0.33	−0.47	−0.53	−0.79	−1.17	−1.96
	Eq. (1)	−0.67	−1.05	−1.06	−1.69	0.46	0.74
TEIS	$\eta_a-0.2$	−1.05	−1.63	−0.67	−1.01	0.97	1.60
	$\eta_b-0.25$	−0.53	−0.81	−0.82	−1.24	not predicted	
Slip factor	Stanitz	1.53	−0.47	1.59	−0.34	1.73	0.12
	Stodola	−2.25	0.47	−2.36	0.34	−2.49	−0.25
	Paeng & Chung	0.29	−0.12	0.29	−0.11	0.30	0.00
Skin friction	k-0.003	1.72	2.56	1.59	2.37	1.73	2.70
	$k+0.003$	−1.58	−2.33	−1.49	−2.25	−1.63	−2.70

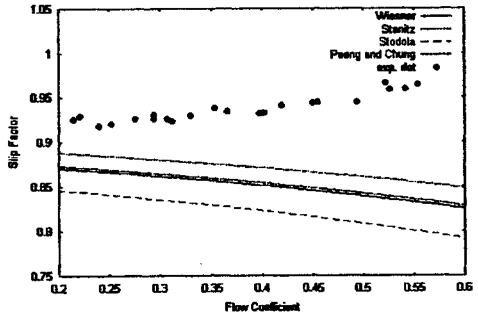

Fig. 16 Slip factor of Eckardt A

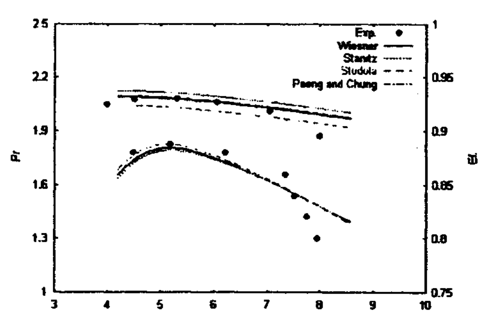

Fig. 17 Effect of slip factor model on Eckardt O

sigcificant effect on pressure ratio and efficiency. The slip factor model has a significant effect on pressure ratio as well, but it has little effect on efficiency. The variation of χ value has a noticeable effect only at the choke point. With this table, we can judge the relative importance of parameters in predicting off-design performance.

5. Conclusion

To gain insight, a detailed sensitivity analysis of empirical models was conducted for a centrifugal compressor, consisting of an impeller and a vaneless diffuser. To analyze the impeller, we adopted the Two-zone model, the TEIS model, and the slip factor model. For the vaneless diffuser, the skin friction coefficient was applied. Our investigation demonstrate the limitations of the present models, and the identification of these limitations may enable us to establish physically more reasonable empirical models.

Based on this examination, the following conclusions are drawn.

Firstly, we discovered that in the Two-zone model, the secondary flow mass fraction has a considerable effect at high mass flow-rates on performance. Also, the recently proposed model, Eq. (1), needs more verification. The secondary flow deviation angle has little or no effect on performance.

Secondly, our experiments with the TEIS model led us to conclude that the first element's effectiveness changes the slope of the performance

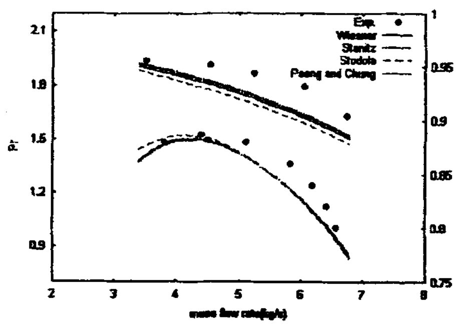

Fig. 18 Effect of slip factor model on Eckardt A

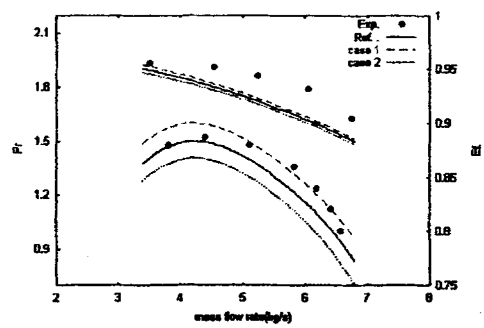

Fig. 20 Effect of k on Eckardt A

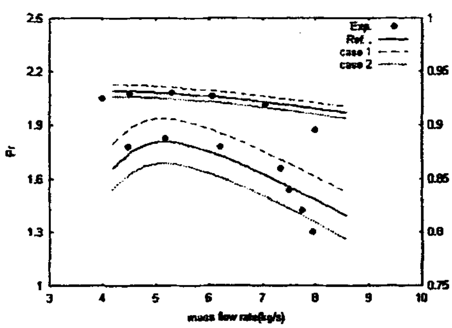

Fig. 19 Effect of k on Eckardt O

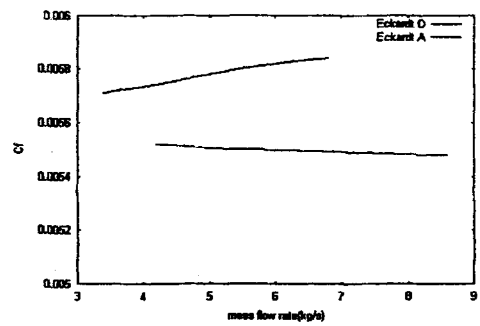

Fig. 21 Effect of k on Cf

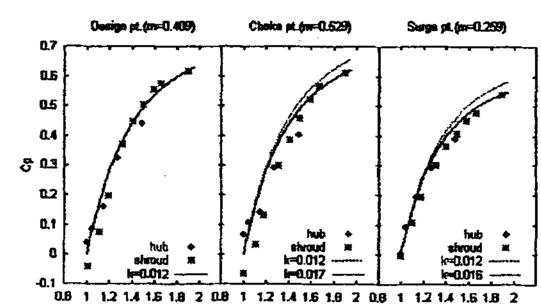

Fig. 22 Pressure recovery coefficient for Cho et al's compressor

curves as well as the stable operating range. The first element is very sensitive to mass flow-rate because it is a variable geometry device which is sensitive to mass flow-rate. Although second element's effectiveness makes the performance curve move up as it increases. It does not influence the stable operating range because the second element is essentially a constant geometry element device which is not sensitive to mass flow-rate.

Thirdly, we further learned that the slip factor model affects the pressure ratio curve substantially, but it has a little effect on the efficiency curve. All the slip factor models' prediction results show a large discrepancy from experimental data for a backswept impeller, perhaps because the slip factor models are overly simplified.

Finally, the skin friction coefficient has an enormous effect on both pressure ratio and efficiency. As we can see in the pressure recovery coefficient curves in the vaneless diffuser, the present practice of setting k in Eq. (6) to a constant value results in inaccurate predictions at off-design conditions.

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